



Answer the following questions

Question 1( 10 marks)

- (a). ( 4 marks) Classify each of the following integral equations as: Volterra or Fredholm integral equation, linear or nonlinear, and homogeneous or non-homogeneous:

i).  $u(x) = x + \int_0^1 (x-t)^2 u(t) dt$

ii).  $u(x) = e^x + \int_0^x t^2 u^2(t) dt$

- (b). ( 4 marks) Classify each of the following integro-differential equations as: Volterra integro-differential equations or Fredholm integro-differential equations. Also determine whether the equation is linear or nonlinear.

i).  $u'(x) = 1 + \int_0^x e^{-2t} u^3(t) dt$  and  $u(0) = 0$

ii).  $u''(x) = \frac{x^2}{2} - \int_0^x (x-t) u^2(t) dt$

- (c) ( 2 marks) Write the homogeneous and non-homogeneous Volterra integral equation of the second kind, describing its terms.

Question 2( 20 marks)

Solve the following Volterra integral equation of the second kind of the convolution type using (a) the Laplace transform method and (b) successive approximation method

$$u(x) = f(x) + \lambda \int_0^x e^{(x-t)} u(t) dt$$

**Question 3( 25 marks)**

- (a) ( 5 marks) Write Volterra integral equations of the first and how can be reduces to Volterra equation of the second kind
- (b) ( 20 marks) Obtain the solution of the following Volterra equation using the series method

$$u(x) = 1 + 2 \sin x - \int_0^x u(t) dt .$$

**Question 4( 45 marks)**

- (a) (5 marks) Write different forms Fredholm integral equations.
- (b) (20 marks) Solve the following Fredholm integral equation by using the successive approximation method

$$u(x) = 1 + \int_0^1 x u(t) dt$$

- (c) (20 marks) Use the successive approximation to solve the following Fredholm integral equation.

$$u(x) = \sin x + \int_0^{\pi/2} \sin x \cos t u(t) dt$$

***With our best wishes***

This exam measures the following ILOs

Question Number	Q1-a	Q3-a				Q1-b	Q1-c	Q4-b			Q3-b	Q4-c	Q2		
<i>Skills</i>	a2-2	a5-1				b2-1	b5-1	b2-2			c4-2	c5-1	c7-2		
	Knowledge& Understanding Skills					Intellectual Skills					Professional Skills				